



## Stability of Electrostatic Waves in a Multi-Ion Permeating Plasma.

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### Abstract

*We have studied the stability of electrostatic waves in a multi-ion, permeating plasma. We consider a neutral solar wind consisting of hydrogen ions and electrons flowing into a pair ion cometary plasma composed of positively and negatively charged oxygen ions and cometary electrons. The dispersion relation obtained is a polynomial equation of order 8. We find that the growth rate of the wave increases with increasing drift velocity of hydrogen ions. It, however, decreases with increasing hydrogen and oxygen ion densities.*

Key words: Electrostatic waves, Multi-ion permeating plasma, Pair ion plasma.

### Introduction

The electrostatic ion cyclotron (EIC) wave instability is a low frequency instability that has been extensively studied because it has one of the lowest thresholds among current driven instabilities. The wave can be driven unstable by electrons drifting parallel to the magnetic field because it has a small, but finite, wave number along the ambient magnetic field even while propagating nearly perpendicular to the magnetic field. Early research on the instability was by Drummond and Rosenbluth [1]; the literature on it was reviewed by Kindel and Kennel [2] and Rasmussen and Schrittwieser [3]. Studies on this wave were also extended to plasmas of other compositions: it was studied in a negative ion plasma using fluid [4] and kinetic analyses [5,6]. Other complexities investigated were the roles of a sheared magnetic field [7] and collisions in a strongly magnetised dusty plasma [8].

Electrostatic noise which was detected by the ICE spacecraft in the vicinity of comet Giacobini-Zinner [9, 10] which exhibited a “low” and “high” frequency structure. The high frequency noise which was electrostatic in nature could extend up to the local electron plasma frequency [11]. In trying to explain the presence of these waves it was realised that the relative motion between the new born ions the background solar wind could be a source of instability for these waves. A new born ion that was often considered was positively charged oxygen. However, negatively charged ions in 3 mass bands of 7-19, 22-65 and 85-110 amu were observed in the coma of comet Halley with negatively charged oxygen ions being conclusively identified [12].

We therefore study the stability of electrostatic waves in a plasma consisting of positively charged oxygen ( $O^+$ ) and negatively charged oxygen ( $O^-$ ) ions. Electrons denoted by ‘ce’ constitute the third component of our cometary plasma. This plasma is impinged upon by a neutral solar wind composed of hydrogen ions ( $H^+$ ) and solar electrons (‘se’). We find that this solar wind can drive electrostatic waves unstable with the growth rate increasing with increasing drift velocities of the hydrogen ions ( $v_{dH}$ ); it decreases with increasing hydrogen ( $n_H$ ) and positively charged oxygen ( $n_{O^+}$ ) ions.

### The Dispersion Relation

We are interested, in this paper, with the stability of electrostatic waves in a cometary plasma composed of positively and negatively charged oxygen ions (denoted respectively by  $O^+$  and  $O^-$ ) and cometary electrons(‘ce’). This plasma is impinged upon by the neutral solar wind composed of hydrogen ( $H^+$ ) and solar electrons(‘se’). The relevant equations are the equation of motion (1), equation of continuity (2) and the Poisson’s equation (3). These are:

$$m_j \left[ \frac{\partial \vec{v}_j}{\partial t} + (\vec{v}_j \cdot \nabla) \vec{v}_j \right] = e_j \left[ \vec{E} + \vec{v}_j \times \vec{B} \right] \quad (1)$$

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \vec{v}_j) = 0 \quad (2)$$

$$\nabla \cdot \vec{E} = 4 \pi \rho \quad (3)$$

In the above ‘j’ indicates the particle species (j= $H, O^+, O^-$ , ‘ce’ or ‘se’),  $m_j$  and  $e_j$  denote the mass and charge of species j while  $n_j$  denote their densities.  $\vec{E}$  and  $\vec{B}$  denote, respectively, the electric and magnetic fields with the ambient magnetic field in the z- direction ( $B_0 \hat{z}$ ). Also the zero order drift is along the z direction ( $v_{dH} \hat{z}$  and  $v_{de} \hat{z}$ );  $\rho$  is the charge density. Assuming a perturbation of the form  $e^{i(k \cdot r - \omega t)}$  we can, using (1) to (3) arrive at a dispersion relation

$$\begin{aligned} & \bar{K} \omega^2 \bar{\omega}_H^{-2} (\omega^2 - \Omega_o^2) (\bar{\omega}_H^{-2} - \Omega_H^2) \\ & - \omega_{pO^+}^2 \left( 1 + \frac{n_{O^-}}{n_{O^+}} \right) \bar{\omega}_H^{-2} (\omega^2 - \Omega_H^2) \left[ \frac{k_z^2}{k^2} (\omega^2 - \Omega_o^2) + \frac{k_x^2}{k^2} \omega^2 \right] \\ & - \omega_{pH}^2 (\omega^2 - \Omega_o^2) \omega^2 \left[ \frac{k_z^2}{k^2} (\bar{\omega}_H^{-2} - \Omega_H^2) + \frac{k_x^2}{k^2} \bar{\omega}_H^{-2} \right] \\ & + \frac{(\omega_{pe}^2)_{se}}{\Omega_e^2} (\omega^2 - \Omega_o^2) (\bar{\omega}_H^{-2} - \Omega_H^2) \frac{k_z^2}{k^2} \left[ \omega^2 + \frac{n_{ce}}{n_{se}} \bar{\omega}_H^{-2} \right] = 0 \end{aligned}$$

.....(4)

In (4)  $\bar{K} = 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} \frac{k_x^2}{k^2}$  and  $\Omega_j = \frac{eB_0}{m_j c}$  is the gyro frequency of species j ('c' is the velocity of light)

while  $\omega_{pj}^2 = \frac{4\pi n_{0j} e^2}{m_j}$  is the square of the plasma frequency with  $n_{0j}$  denoting the equilibrium density of

species j. Also  $k_x$  and  $k_z$ , denote respectively, the wave vector perpendicular and parallel to the ambient

magnetic field.  $\bar{\omega}_H$  is the Doppler shifts frequency defined by  $\bar{\omega}_H = \omega - k_z v_{dH}$ . The only assumption

made in deriving (4) is  $\bar{\omega}_e \ll \Omega_e$  where  $\bar{\omega}_e = \omega - k_z v_{de}$ .

When expanded, equation(4) results in a polynomial equation of degree 8. However, since the coefficients are very lengthy, they will not be given here. It may be noted at this point that the dispersion relation for low frequency electromagnetic wave is a polynomial equation of degree 9 [13].

### Results

Equation(4), which is a polynomial equation of degree 8 was solved by the Laugerre's method[14]. The parameters chosen were [15] hydrogen density  $n_{oH} = (n_o)_{se} = 4.9\text{cm}^{-3}$ ,  $n_{o+} = 0.3\text{cm}^{-3}$  and  $n_{o-} = 0.05\text{cm}^{-3}$ . The cometary electron density  $(n_o)_{ce}$  was calculated from the charge neutrality condition. The hydrogen drift velocity  $v_{dH} = 400 \times 10^5 \text{ cms}^{-1}$ ; the electron drift velocity does not enter (4).

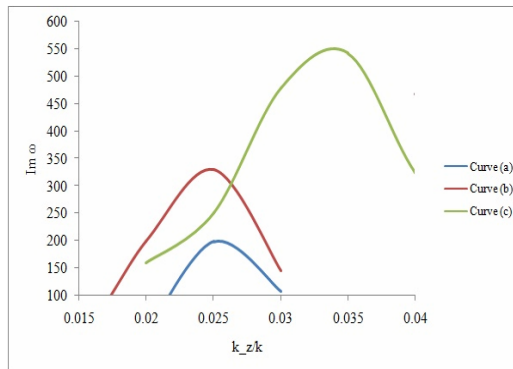


Figure (1): plot of  $\text{Im } \omega$  versus  $\frac{k_z}{k}$  as function of  $v_{dH}$

Figure 1 is a plot of the imaginary part of the frequency  $\omega$  ( $\text{Im } \omega$ ) versus  $\frac{k_z}{k}$  for  $n_H = 4.9 \text{ cm}^{-3}$ ,  $n_{o+} = 0.3 \text{ cm}^{-3}$  and  $n_{o-} = 0.05 \text{ cm}^{-3}$  as a function of the drift velocity  $v_{dH}$  of the hydrogen ions. Curve (a) is for  $v_{dH} = 400 \text{ km s}^{-1}$ , curve (b) for  $v_{dH} = 600 \text{ km s}^{-1}$  and curve (c) for  $800 \text{ km s}^{-1}$ . We find that the growth rate increases with increasing drift velocity of the hydrogen ions.

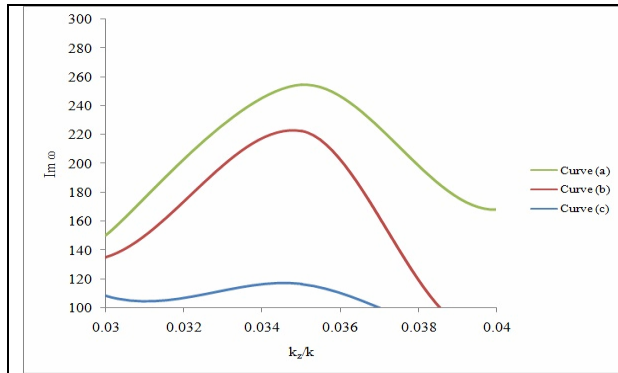


Figure 2 plot of  $\text{Im } \omega$  versus  $\frac{k_z}{k}$  as function of  $n_{O^+}$

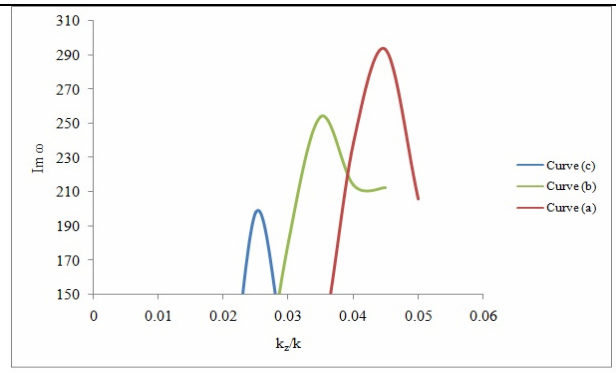


Figure 3 plot of  $\text{Im } \omega$  versus  $\frac{k_z}{k}$  as function of  $n_H$

The influence of the density of positively charged oxygen ions on the wave growth is studied next. Figure (2) is a plot of  $\frac{k_z}{k}$  versus  $\text{Im } (\omega)$  for  $v_{dH} = 400 \text{ km s}^{-1}$ ,  $n_{oH} = 4.9 \text{ cm}^{-3}$  and  $n_{O^-} = 0.05 \text{ cm}^{-3}$ . Curve (a) is for  $n_{O^+} = 0.1 \text{ cm}^{-3}$ , curve (b) is for  $n_{O^+} = 0.2 \text{ cm}^{-3}$  and curve (c) is for  $n_{O^+} = 0.3 \text{ cm}^{-3}$ . We find that the growth rate decreases with increasing oxygen densities.

Figure 3 complements figure 2 and depicts the variation of  $\frac{k_z}{k}$  versus  $\text{Im } \omega$  as a function of hydrogen ion densities. Curve (a) is for  $n_H = 4.95 \text{ cm}^{-3}$ , curve (b) is for  $n_H = 4.5 \text{ cm}^{-3}$  and curve (c) is for  $n_H = 4 \text{ cm}^{-3}$ ; the other parameters for the figure are  $n_{O^+} = 0.3 \text{ cm}^{-3}$ ,  $n_{O^-} = 0.05 \text{ cm}^{-3}$  while  $v_{dH} = 400 \text{ km s}^{-1}$ . Similar to figure (2) the growth rate decreases with increasing hydrogen densities.

## Conclusions

We have studied the stability of electrostatic waves in a cometary plasma composed of positively and negatively charged oxygen ions and cometary electrons. This plasma is impinged upon by the solar wind composed of hydrogen ions and solar electrons. The dispersion relation is a polynomial equation of order 8; the growth rate increases with increasing velocity of the solar wind. The growth rate, however, decreases with increasing hydrogen and positively charged oxygen ion densities. We thus find that the electrostatic wave can be driven unstable even when propagating in a permeating plasma.

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## References

- [1] W.E. Drummond and M.N. Rosenbluth, "Anomalous Diffusion Arising from Microinstabilities in a Plasma", *Phys. Fluids*, vol.5, pp.1507 , 1962.
- [2] J.M. Kindel and C.F.Kennel, "Top side current instabilities", *J.Geophys. Res.*, vol. 76, pp.3055, 1971.
- [3] J.J. Rasmussen and R.W. Schrittwieser, "On the current-driven electrostatic ion-cyclotron instability: a review", *Plasma Science, IEEE Transactions on Plasma Science*, vol. 19, pp.457, 1991.
- [4] N. D' Angelo and R. L. Merlino, "Electrostatic ion-cyclotron waves in a plasma with negativeions", *IEEE Transactions on Plasma Science*, vol. 17, pp.285, 1986.
- [5] V. W.Chow, M. Rosenberg, "A note on the electrostatic ion cyclotron instability in dusty plasmas: comparison with experiment", *Planetary and Space Science*, vol. 44, pp.465, 1996.
- [6] M.J. Kurian, S. Jyothi, S.K.Leju, Molly Isaac, Chandu Venugopaland G. Renuka, "Stability of electrostatic ion cyclotron waves in a multi-ion plasma", *Pramana – J. Phys.*, vol. 73, pp.1111, 2009.
- [7] H. Jalori and A.K. Gwal, "Role of magnetic shear on the electrostatic current driven ion-cyclotron instability in the presence of parallel electric field", *Pramana – J. Phys.*, vol.56, pp.779 , 2001.
- [8] M. Rosenberg and P.K. Shukla, "Instability of obliquely propagating dust waves in a collisional highly magnetized plasma", *Journal of Plasma Physics*, vol. 73, pp 189-197 , 2007.
- [9] F.L. Scarf, F.V.Coroniti, C.F. Kennel, D.A. Gurnett, W.H. Ip and E.J.Smith, "Plasma wave observations at comet Giacobini-Ziner", *Science*, vol.232, pp.377, 1986.
- [10] I.G. Richardson, K.P. Wenzel, S.W.H. Cowley, F.L. Scarf, E.J. Smith, B.T. Tsurutani, and R.J. Hynds (1989), "Correlated plasma wave, magnetic field, and energetic ion observations in the ion pickup region of comet Giacobini-Zinner", *Journal of Geophysical Research: Space Physics* (1978–2012), vol.94(A1), pp. 49-59, 1989.
- [11] A. L. Brinca and B. T. Tsurutani, "The oblique behavior of low-frequency electromagnetic waves excited by newborn cometary ions" *J. Geophys. Res.*, vol. 94, pp. 3-14, 1989.
- [12] P. Chaizy, H. Reme , J.A. Sauvaud, C. d'Uston , R.P.Lin , D.E. Larson, D.L. Mitchell, K.A. Andersen, C.W. Carlson, A. Korth and D.A. Mendis, "Negative ions in the coma of comet Halley", *Nature*, vol. 349, pp.393, 1991.
- [13] Chandu Venugopal, M.J. Kurian, E. Savithri Devi, P.J. Jessy, C.P. Anilkumar, and G. Renuka, "Low frequency electromagnetic waves in a multi-ion plasma", *Indian J. Phys.*, vol. 84, pp.319-324, 2010.



- [14] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, “Numerical recipes in FORTRAN: The art of scientific computing”, 2nd edn Foundation Books, New Delhi, 1993.
- [15] A.L. Brinca and B.T. Tsurutani, “Unusual characteristics of electromagnetic waves excited by cometary new born ions with large perpendicular energies”, *Astron. Astrophys.*, vol. 187, pp.311, 1987